Daniel Hug and Wolfgang Weil: **Lectures on Convex Geometry**. Graduate Texts in Mathematics, 286. Springer, Cham, [2020], 2020. xviii+287 pp. ISBN: 978-3-030-50180-8; 978-3-030-50179-2

Typos, comments and supplements

- Page 5, Remark 1.5: See Excercise 1.1.4.
- Page 9, Exercise 1.1.5. Additional task: Show by an example that the assumption that A be closed cannot be dispensed with.
- Page 11, Exercise 1.1.18. Hint: See Chapter 3 of the PhD thesis by Ina Voigt, Voronoizellen diskreter Punktmengen, TU Dortmund, 2008. See also ArXiv.0811.1525, Corollary 3.1 and Example 3.1. Further information is contained in I. Voigt and S. Weis. Polyhedral Voronoi cells. Beiträge Algebra Geom. 51 (2010), no. 2, 587–598.
- Page 20, line 15. Typo: $z_\varepsilon := (1-\varepsilon)x + \varepsilon y$.
- Page 21, line 11. Insert: If $y \neq x$, by Theorem 1.11 (c) we obtain ...
- Page 30, Proof of Theorem 1.19. Instead of assuming w.l.o.g. that $x_1, \dots, x_m \in F$, one can work with $I := \{i \in \{1, \dots, k\} : x_i \in F\}$.
- Page 30, line 19. Typo: $r = 2(n-k)$.
- Page 31, Exercise 1.4.3. It is not needed that A is convex.
- Page 33, Remark 1.24. ...if and only if $A \neq \emptyset$ does not ...
- Page 36, line -6. Typo: $\|z - \bar{x}\| > \max\{r, (r^2 - \delta^2/(2\delta)\}$
- Page 37, Exercise 1.5.1. Let $\emptyset \neq A \subset \mathbb{R}^n$ be closed and convex.
- Page 39, Exercise 1.5.8. By a semi-flat we mean a half-flat, that is a halfspace within a flat.
- Page 49, line 4. The inequality can be replaced by an equality.
- Page 52, line -7. We may also choose $\alpha \in [0, 1]$.
- Page 53, line 11. Typo: ...for $z \in x + U$.
- Page 58, line 5. The entries of the secondary diagonal of the matrix should be $-\frac{y}{x^2}$.
- Page 59, line 8. Let $x \in \text{int dom } f$.
- Page 67, line 12. Of course, the inequality can be replaced by an equality.
- Page 74, line 14. Insert: ...of a convex body K as a ...
- Page 78, line 1. Note that since $(K_k)_{k\in\mathbb{N}}$ is a Cauchy sequence, there is a cube W such that $K_k \subset W$ for $k \in \mathbb{N}$.
- Page 95, line 11. In other words, $N(P_1, \dots, P_k)$ is the set of all $u \in \mathbb{S}^{n-1}$ such that $(P_1 + \cdots + P_k)(u)$ is a facet of $P_1 + \cdots + P_k$.
- Page 95, after Definition 3.4. Note that by induction it follows that $V^{(n)}(P, \dots, P) = V^{(n)}(P)$.
- Page 96, line 3. Typo: $E \subset \mathbb{R}^n$.
- Page 98, line -11 to -9: See Exercise 1.4.6 and Exercise 1.5.5.
- Page 99, line 12. If $\tan \gamma(u, v) = \infty$, we interpret $\frac{1}{\tan \gamma(u,v)}$ as 0.

- Page 100, Example 3.1. In the example, we assume that K_i is a polytope. We also use the preceding comment on “Page 95, after Definition 3.4”.
- Page 102, line 1. Replace “representation of f” by “polynomial expansion of $f(\alpha_1 P_1, \ldots, \alpha_n P_n)$ via (3.7)”.
- Page 109, line 8. Insert “)” before the full stop.
- Page 110, lines -3 to -5. The constant c can be any real number. (This is what the proof shows.)
- Page 113, Exercise 3.3.3(b). The problem has been resolved (in a strengthened form) in F.A. Bartha, F. Bencs, K.J. Böröczky and D. Hug. Extremizers and stability of the Betke–Weil inequality. Michigan Math. J. **74** (2024), 45–71.
- Page 114, Exercise 3.3.9. Insert a full stop at the end of the first sentence.
- Page 120, line -2. Typo: …(compare Exercise 3.3.1).
- Page 120, line 1. See also Exercise 3.5.1.
- Page 147, line 4. …and mixed volumes of the faces with the same …
- Page 157, line -4. Typo: $x = t/2$.
- Page 158, line 13. Typo: Integration domain is $\mathbb{S}^{n-1}$.
- Page 163, line -10 to bottom of the page. Avoid the use of k_0 and write $k_0(\varepsilon)$ throughout.
- Page 163, lines -6, -5. Replace the equality sign by “$\leq$”.
- Page 179, line -1. Here we assume that $x \in \mathbb{S}^{n-1}$.
- Page 185, line -7. Here we use Exercise 2.3.5 or Lemma 4.2.
- Page 188, line -8, -7. Replace μ by ν_1 and ρ by ν_2.
- Page 189, line -3. The statement of Lemma 4.7 is formally weaker, but this special case …
- Page 192, line 17. Here we require that $n \geq 3$.
- Page 196, line -3. Replace “notion” by “notation”.
- Page 203, line -9. Typo: If $m \in \mathbb{N}$ …
- Page 204, line 10. $\triangle_0 :=$ conv$\{v, v - v_1, \ldots, v - v_n\}$.
- Page 209, line 2. Typo: LU$_n$.
- Page 209, line 7. Typo: $\mathbb{R}^{n,n}$
- Page 212, line 15, 16. Delete the sentence “Thus …topological space.”
- Page 212, line -7. The sentence “Then the map $\varphi(g, \cdot) : X \to X$ is a homeomorphism.” should be replaced by “Then the map $\varphi(g, \cdot) : X \to X$ is a homeomorphism for all $g \in G$.” and exchanged in order with the preceding sentence.
- Page 212, line -3. Write: …stabilizer subgroup of $x \in X$.
- Page 213, line 3. Insert: Note that if $A \in \mathcal{B}(X)$ and $g \in G$, then $gA \in \mathcal{B}(X)$.
- Page 215, line -9. Typo: $\nu_k(dU)$.
- Page 223, lines 12, 13: Factor β_{nik} on the right-hand side is missing.

- Page 225, line 12. Typo: Bracket “(” is missing.
- Page 233, line 10 and statement of Lemma 5.7. Add the requirement that $\varphi(\emptyset) := 0$.
- Page 233, statement of Lemma 5.7. The exceptional set $\mathrm{G}^*(n)$ should be defined as the set of all $g \in \mathrm{G}(n)$ for which $K \cap gL \neq \emptyset$ and such that K and gL can be separated by a hyperplane.
- Page 233, line -7: Replace $g \in \mathrm{G}^*(n)$ by $g \in \mathrm{G}(n) \setminus \mathrm{G}^*(n)$.
- Page 233, Lemma 5.7. The purpose of the lemma is to confirm that the integrand of the principal kinematic formula (Theorem 5.12), that is, the map $g \mapsto \varphi(K \cap gL)$, $g \in \mathrm{G}(n)$, is measurable, whenever φ is measurable (at least for $\varphi = V_i$). In fact, this follows more generally from the following lemma.

  Let $\mathcal{K}^n_* := \mathcal{K}^n \cup \{\emptyset\}$ be endowed with the $[0,\infty]$-metric d_* with $d_*(\emptyset,\emptyset) := 0$, $d_*(\emptyset, K) = d_*(K,\emptyset) := \infty$ if $K \in \mathcal{K}^n$ and $d_* = d$ on $\mathcal{K}^n \times \mathcal{K}^n$. Hence d_* extends the Hausdorff metric d so that $\emptyset$ is an isolated point in $\mathcal{K}^n_*$. A set $\mathcal{A} \subset \mathcal{K}^n_*$ is open with respect to the topology induced by d_* if and only if $\mathcal{A} \cap \mathcal{K}^n$ is open with respect to the topology induced by d. Moreover, for the respective Borel sets we have $\mathcal{B}(\mathcal{K}^n_*) = \mathcal{B}(\mathcal{K}^n) \cup \{\mathcal{M} \cup \{\emptyset\} : \mathcal{M} \in \mathcal{B}(\mathcal{K}^n)\}$ or $\mathcal{B}(\mathcal{K}^n_*) = \{\mathcal{M} \subset \mathcal{K}^n_* : \mathcal{M} \cap \mathcal{K}^n \in \mathcal{B}(\mathcal{K}^n)\}$.

  For a compact set $C \subset \mathbb{R}^n$ we define $\mathcal{K}^C_* := \{K \in \mathcal{K}^n_* : K \cap C = \emptyset\}$. Let $\mathcal{H}^n$ denote the σ-algebra on $\mathcal{K}^n_*$ generated by the sets $\mathcal{K}^C_*$, where $C \subset \mathbb{R}^n$ is compact.

  Lemma. (a) The σ-algebra $\mathcal{H}^n$ equals $\mathcal{B}(\mathcal{K}^n_*)$.

  (b) The map $f : \mathcal{K}^n_* \times \mathcal{K}^n_* \to \mathcal{K}^n_*$, $(K, L) \mapsto K \cap L$, is Borel measurable.

  Proof. (a) For each compact set $C \subset \mathbb{R}^n$, the set $\mathcal{K}^C_*$ is open with respect to the topology induced by d_*, hence the inclusion $\mathcal{H} \subset \mathcal{B}(\mathcal{K}^n_*)$ follows.

  We turn to the reverse inclusion. Since $(\mathcal{K}^C_*, d_*)$ is a separable metric space, it suffices to show that
  $$B_*(K,\varepsilon) = \{L \in \mathcal{K}^n_* : L \subset K + \varepsilon B^n, K \subset L + \varepsilon B^n\} \in \mathcal{H}^n,$$
  for each $K \in \mathcal{K}^n_*$ and $\varepsilon \in (0,\infty)$.

  - If $K = \emptyset$, then
    $$B_*(\emptyset,\varepsilon) = \{\emptyset\} = \bigcap_{\ell \in \mathbb{N}} \mathcal{K}^{\ell B^n}_* \in \mathcal{H}^n.$$
  - Let $K \in \mathcal{K}^n$. There exists a sequence of compact sets $B_j \subset \mathbb{R}^n$, $j \in \mathbb{N}$, such that
    $$\bigcup_{j \geq 1} B_j = \mathbb{R}^n \setminus (K + \varepsilon B^n),$$
    hence
    $$\{L \in \mathcal{K}^n_* : L \subset K + \varepsilon B^n\} = \bigcap_{j \geq 1} \mathcal{K}^{B_j}_* \in \mathcal{H}^n.$$
    Moreover, let $D \subset K$ be a countable, dense subset. Then
    $$K \not\subset L + \varepsilon B^n \iff \exists x \in D : B^n(x,\varepsilon) \cap L = \emptyset \iff \exists x \in D : L \in \mathcal{K}^{B^n(x,\varepsilon)}_*,$$
    hence
    $$\{L \in \mathcal{K}^n_* : K \subset L + \varepsilon B^n\} = \bigcap_{x \in D} \left(\mathcal{K}^n_* \setminus \mathcal{K}^{B^n(x,\varepsilon)}_*\right) \in \mathcal{H}^n.$$
    Thus we get $B_*(K,\varepsilon) \in \mathcal{H}^n$.

(b) We claim that $f^{-1}(\mathcal{K}_*^C)$ is open for each compact set $C \subset \mathbb{R}^n$. Suppose not. Then there are a compact set $C \subset \mathbb{R}^n$, $K, L \in \mathcal{K}_*^n$ with $K \cap L \cap C = \emptyset$, and $(K_i, L_i) \in \mathcal{K}_*^n$ with $K_i \to K$, $L_i \to L$, as $i \to \infty$, with $K_i \cap L_i \cap C \neq \emptyset$ for $i \in \mathbb{N}$. Since $K_i \to K$ and $K_i \neq \emptyset$, it follows that $K \neq \emptyset$ (by the definition of d_*). Similarly, we have $L \neq \emptyset$. Let $x_i \in K_i \cap L_i \cap C$ for $i \in \mathbb{N}$. Since C is compact, there is an infinite subset $I \subset \mathbb{N}$ such that $x_i \to x_0 \in C$ as $i \to \infty$, $i \in I$. Since $x_i \in K_i$ and $K_i \to K$ (in fact with respect to d), it follows that $x \in K$, and by the same argument also $x \in L$. This shows that $x \in K \cap L \cap C \neq \emptyset$, a contradiction. This proves the claim.

The asserted measurability now follows as usual from (a). □

- Page 239, Exercise 1.1.3. Replace m by k to be consistent with the statement of the exercise.
- Page 240, Exercise 1.1.8. Replace "⊃" by "⊂" and vice versa.
- Page 244, Exercise 1.3.3, line -7. Write: $\alpha_i \in \mathbb{R}$.
- Page 249, Exercise 1.5.3, line 7. Typo: $\dots x \in (y, z) \subset N \subset A$.
- Page 256, Exercise 3.1.8, line 20. Refer to Exercise 3.1.5(a).
- Page 259, line 13. Replace the upper index j by j_0.
- Page 261, Exercise 3.4.2, line -3. Insert: For the proof, we can assume that $V(K) > 0$.
- Page 263, Exercise 3.4.11, line 7. Typo: …compact subsets of $\mathbb{R}^n$.
- Page 263, Exercise 3.4.13, line 4. Typo: The general case then …
- Page 282, Reference [20]. Update of the bibliographic data: K. Böröczky and D. Hug. A reverse Minkowski-type inequality. Proc. Amer. Math. Soc. 148 (2020), 4907–4922.
- Page 284, Reference [86]. Update of the bibliographic data: Y. Shenfeld and R. van Handel. The extremals of Minkowski's quadratic inequality. Duke Math. J. 171 (2022), no. 4, 957–1027.